Matrices

P K Subhadra Devi

Source : MATHEMATICAL METHODS FOR PHYSICISTS 6e. George B. Arfken & Hans J. Weber

Gauss Elimination

Solve

$$3x + 2y + z = 11$$
$$2x + 3y + z = 13$$
$$x + y + 4z = 12.$$

The determinant of the inhomogeneous linear equations (3.19) is 18, so a solution exists.

For convenience and for the optimum numerical accuracy, the equations are rearranged so that the largest coefficients run along the main diagonal (upper left to lower right). This has already been done in the preceding set.

The Gauss technique is to use the first equation to eliminate the first unknown, x, from the remaining equations. Then the (new) second equation is used to eliminate y from the last equation. In general, we work down through the set of equations, and then, with one unknown determined, we work back up to solve for each of the other unknowns in succession.

Dividing each row by its initial coefficient, we see that Eqs. (3.19) become

$$x + \tfrac{2}{3}y + \tfrac{1}{3}z = \tfrac{11}{3}$$
$$x + \tfrac{3}{2}y + \tfrac{1}{2}z = \tfrac{13}{2}$$
$$x + y + 4z = 12.$$

Now, using the first equation, we eliminate x from the second and third equations:

$$\begin{aligned} x + \tfrac{2}{3}y + \tfrac{1}{3}z &= \tfrac{11}{3} \\ \tfrac{5}{6}y + \tfrac{1}{6}z &= \tfrac{17}{6} \\ \tfrac{1}{3}y + \tfrac{11}{3}z &= \tfrac{25}{3} \end{aligned}$$

and

$$\begin{aligned} x + \tfrac{2}{3}y + \tfrac{1}{3}z &= \tfrac{11}{3} \\ y + \tfrac{1}{5}z &= \tfrac{17}{5} \\ y + 11z &= 25. \end{aligned}$$

Repeating the technique, we use the new second equation to eliminate y from the third equation:

$$\begin{aligned} x + \tfrac{2}{3}y + \tfrac{1}{3}z &= \tfrac{11}{3} \\ y + \tfrac{1}{5}z &= \tfrac{17}{5} \\ 54z &= 108, \end{aligned}$$

or

$$z = 2.$$

Finally, working back up, we get

$$y + \tfrac{1}{5} \times 2 = \tfrac{17}{5},$$

or

$$y = 3.$$

Then with z and y determined,

$$x + \tfrac{2}{3} \times 3 + \tfrac{1}{3} \times 2 = \tfrac{11}{3},$$

and

$$x = 1.$$

The technique may not seem so elegant as Eq. (3.18), but it is well adapted to computers and is far faster than the time spent with determinants.

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Gauss Jordan Elimination

A variation of this progressive elimination is known as Gauss–Jordan elimination. We start as with the preceding Gauss elimination, but each new equation considered is used to eliminate a variable from **all** the other equations, not just those below it. If we had used this Gauss–Jordan elimination, Eq. (3.23) would become

$$\begin{aligned} x + \tfrac{1}{5}z &= \tfrac{7}{5} \\ y + \tfrac{1}{5}z &= \tfrac{17}{5} \\ z &= 2, \end{aligned} \tag{3.24}$$

using the second equation of Eqs. (3.22) to eliminate y from both the first and third equations. Then the third equation of Eqs. (3.24) is used to eliminate z from the first and second, giving

$$\begin{aligned} x \qquad &= 1 \\ y \quad &= 3 \\ z &= 2. \end{aligned} \tag{3.25}$$

Matrices are square or rectangular arrays of numbers that define linear transformations, such as rotations of a coordinate system. As such, they are linear operators. Square matrices may be inverted when their determinant is nonzero. When a matrix defines a system of linear equations, the inverse matrix solves it. Matrices with the same number of rows and columns may be added and subtracted. They form what mathematicians call a ring with a unit and a zero matrix. Matrices are also useful for representing group operations and operators in Hilbert spaces.

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